Creating and Using Videos for Teaching Math

Suggestions from the Field

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Introduction

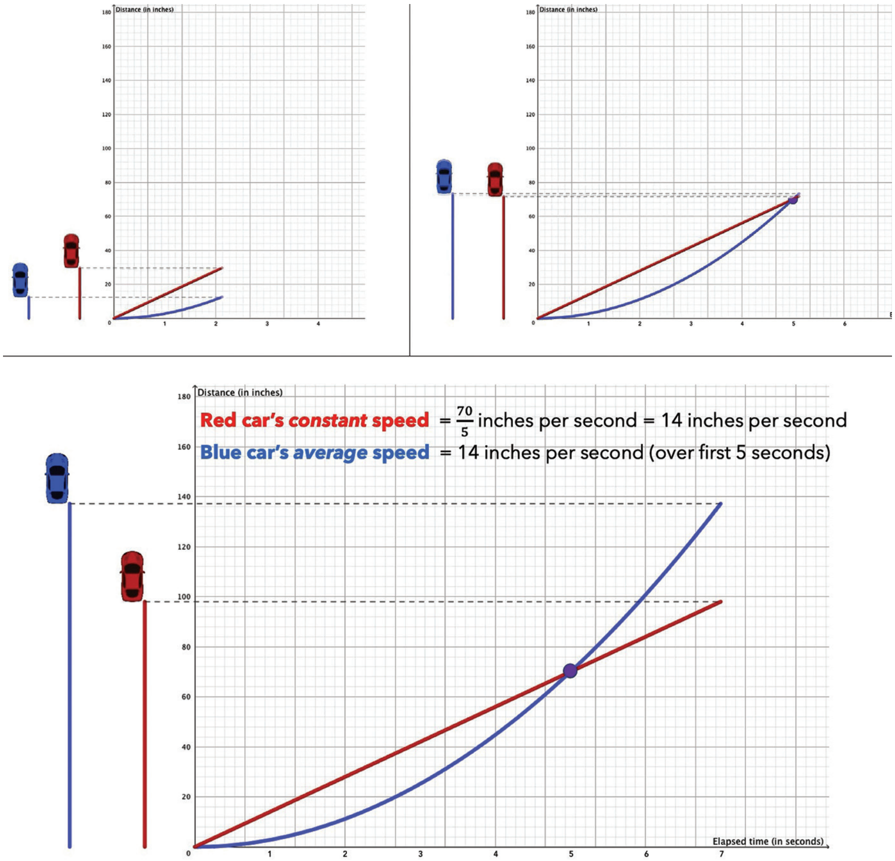
The pivot to online instruction during the spring 2020 academic term required many instructors to try new (to them) methods of instruction and to seek resources to support these methods. Instructional videos are one such resource that can be incorporated in in-person, online, and blended instruction to harness the potential power of flipped pedagogy.

Over the past three years, the NSF-sponsored Calculus Videos Project (calcvids.org/) has created and refined over 50 instructional videos for first-semester college calculus and the team members have been working on research-based curriculum development in undergraduate mathematics for a combined 46 years. In this article, we will describe several aspects to consider when designing instructional videos, present tips for creating videos, and share suggestions for incorporating videos into instruction.

Video Formats

We have observed three basic formats for instructional videos. In the first format, the instructor appears in front of a whiteboard or glass board or uses a tablet or document camera and narrates the lesson while writing on the board/screen. The second format consists of static slides with the instructor narrating what appears on the slides. Both of these formats mimic a traditional lecture.

The third video format more directly takes advantage of the affordances of the video medium by reducing the amount of onscreen text and incorporating a wide variety of visuals and animations. For



Screenshots of a video describing average speed in terms of constant speed.

the risk of failing to support subsequent formal understandings. In our own videos, we have attempted to balance correctness and intuition by adopting precision as a key design criterion. By this, we mean developing the mathematical concepts in a way that leverages students' intuitions while targeting a particular mathematical understanding that is consistent in presentation within and across videos and can support future learning of related concepts.

For example, in the Calcvids videos that introduce the concept of average rate of change, we define average rate as the [constant] rate of change depicted by the slope of a linear function that has the same change in output values over the same interval the average rate is calculated. This definition is illustrated in the screenshots from the video that shows the speeds of two cars, one of which travels at a constant speed. By grounding the concept of average rate of change in terms of the already-developed (in our videos) concept of constant rates of change, we can avoid relying solely on a formula and, simultaneously, set the stage for the introduction of instantaneous rates of change.

example, instructors can visually represent connections between concepts, include dynamic representations from GeoGebra, Desmos, and Wolfram Player, or demonstrate interactions with other software tools. In addition to the Calcvids videos, other examples of this format include 3Blue1Brown (www.3blue1brown.com/) and Socratica (www.socratica.com/).

Suggestions for Video Design

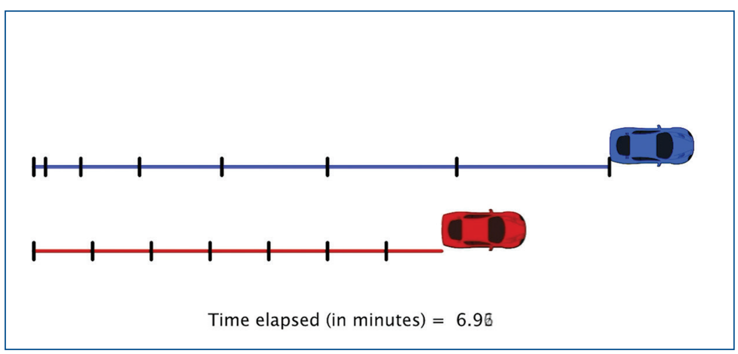
During the process of designing our videos, we have found it essential to focus on the interaction between three aspects: the mathematical content, multimedia learning recommendations, and how students think about and learn mathematical concepts. Below, we describe each aspect and provide an example.

Mathematical Content

It is important for mathematics in a video to be correct (and there are many videos online that contain mistakes). However, a presentation that is too formal risks confusing and alienating students. Conversely, merely intuitive presentations run

Student Thinking and Learning

In addition to focusing on the concepts from a mathematician's perspective, we believe it is important to design videos that take into account what we know about how students think about and learn mathematical concepts. For example, research on students' functional thinking suggests that it is important for students to think about rates of change in terms of coordinating amounts of change in quantities (e.g., "Applying covariational reasoning while modeling dynamic



Screenshot of a video showing amounts of change in distance for one-second intervals of time.

events: A framework and a study," by Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). Thus, in the Calcvids videos about average rate of change, we frame examples in terms of amounts of change of distance, amounts of change of time, and the ways in which these quantities co-vary. This is illustrated in the screenshot below, which shows amounts of change in distance for one-second intervals of time for two racing cars.

There is a great deal of research available that describes student thinking and learning about specific mathematical concepts. One place to start is the MAA publication *Making the Connection: Research and Teaching in Undergraduate Mathematics Education*, edited by Marilyn Carlson and Chris Rasmussen (2008), which describes research on student thinking about variables, change, function, accumulation, infinity, limit, number representations, proof construction, group theory, and differential equations, as well as cross-cutting themes such as definitions, worked examples, and problem solving. The SIGMAA on Research in Undergraduate Mathematics Education is another good resource for identifying relevant research on student thinking and learning about mathematics. In addition to their annual conference and sessions at Mathfest and the Joint Mathematical Meetings, the SIGMAA maintains an active mailing list where faculty can discuss research and its implications for practice.

Multimedia Design

In addition to considering mathematics and student thinking when developing instructional resources, we can improve videos by incorporating research about learning from multimedia. One notable resource is the book Multimedia Learning by Richard Mayer (2009), which describes twelve research-based principles for designing effective multimedia presentations; these principles are also summarized on numerous websites, such as Principles of Multimedia Learning (ctl.wiley.com/principles-of-multimedia-learning/). For example, the coherence principle states that removing extraneous words, images, animations, and sounds supports learning. Although such a recommendation might sound obvious, we have found that it is tempting to include a snazzy visual or to add additional details even when they are not directly tied to the learning goals for a particular video.

The Interaction Between Content, Learning, and Multimedia Design

We believe that it is important to consider the interaction between content, student learning, and multimedia design when designing instructional videos for mathematical concepts. For example, when designing our videos for average rate of change, we needed to think about various ways to conceptualize average rate of change, the alignment between these various conceptions and students' ways of thinking, and the various ways we might represent these aspects—including ways to visualize the dynamic relationships between the quantities.

Practical Considerations for Creating Videos

We have found that making videos is incredibly time-consuming. Our own videos took three years of intense collaboration, testing with students, and revisions—and we are still finding ways to improve them! Thus, if high-quality videos are already available for a topic, we suggest you consider using those videos when possible. Similarly, we suggest making use of any resources already available—for example, the Calcvids project has made its PowerPoint files available on calcvids.org.

If your institution has asked you to create videos or you are interested in creating your own videos, we suggest collaborating with colleagues to support discussion of the mathematics, student thinking, and multimedia aspects and to distribute the work of creating the video materials.

Here are a few practical suggestions for creating video materials:

- Familiarize yourself with the various technology options before jumping in. There are dedicated apps and powerful software available for creating videos. However, you can make a good video with basic presentation software (such as PowerPoint, Keynote, and OpenOffice Impress) by incorporating audio, video, and images, using the built-in animations, and using a touchscreen or pen interface to add drawings.
- Videos should be roughly 5–10 minutes long, so it might be necessary to split the presentation of mathematical concepts and procedures across multiple videos.
- Begin by making a storyboard. This will help you align the mathematics and student thinking with what will appear on screen and will help organize the ideas.
- Write a script to go with the storyboard. This will save you time in the long run.
- As you write the storyboard and script, think of ways you might use the video medium to promote learning that might be different from a standard classroom presentation.
- Record narration in a quiet environment when possible. Using an external microphone with a cardioid polar pattern can help mitigate surrounding noise.
- Don't expect your videos to be perfect!

Suggestions for Incorporating Videos into Instruction

It's also important to consider how to structure the video-watching process and to integrate it into instruction. For example, our own research has shown that students often don't recognize when they don't understand something, and might not take advantage of the ability to pause or rewind the video. Consequently, merely asking students to watch the video is unlikely to lead to substantial learning. Below, we describe a framework for breaking down the video-watching process; the Calcvids website has additional suggestions from instructors who have participated in our research and used our instructional videos.

The video-watching process can be broken down into three phases: the "launch," the "watch," and the "debrief."

- In the "launch," students engage with the concepts or aspects of the video prior to watching it. This phase might entail previewing the concept in class or having students work on a problem that motivates the concept; the solution to this problem could also appear in the video itself.
- In the "watch," students watch and interact with the video. This phase could include taking and submitting notes or writing and submitting questions they had. If the video includes a demonstration such as a Desmos or GeoGebra app, the students might also interact with and write about the app.
- In the "debrief," students share their learning and discuss the concepts in class. This might take the form of a short quiz, discussing the questions students submitted, or having students work on problems that were used in the video.

The research literature currently says very little about best practices for incorporating videos into instruction, and this is a great opportunity for the community to systematically explore this research opportunity.

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